

Complex Numbers – Important Points & Formulae (MHT-CET)

1. Definition

- A complex number is written as: $z = a + ib$
- $a = \text{Real part } \text{Re}(z)$, $b = \text{Imaginary part } \text{Im}(z)$
- $i = \sqrt{-1}$, and $i^2 = -1$

2. Powers of i

- $i^1 = i$
- $i^2 = -1$
- $i^3 = -i$
- $i^4 = 1$
- Cycle repeats every 4 $\rightarrow i^n = i^{(n \bmod 4)}$

3. Conjugate of Complex Number

- If $z = a + ib$ then conjugate $z^* = a - ib$
- $z + z^* = 2a$
- $z - z^* = 2ib$
- $z \cdot z^* = a^2 + b^2$

4. Modulus

- $|z| = \sqrt{a^2 + b^2}$
- $|z|^2 = z \cdot z^*$
- $|z^* z^*| = |z^*| |z|$

5. Argument

- $\arg(z) = \theta$
- $\theta = \tan^{-1}(b/a)$

6. Polar Form

- $z = r (\cos\theta + i \sin\theta)$
- $r = |z|$
- $\theta = \text{argument}$

7. De Moivre's Theorem

- $(\cos\theta + i \sin\theta)^n = \cos(n\theta) + i \sin(n\theta)$

8. Roots of Complex Numbers

- $z^k = r^{1/n} [\cos((\theta + 2k\pi)/n) + i \sin((\theta + 2k\pi)/n)]$
- $k = 0, 1, 2, \dots, n-1$

9. Argand Plane

- Complex number represented as point (a,b)
- X-axis \rightarrow real part
- Y-axis \rightarrow imaginary part
- Distance from origin = modulus

10. Important Identities

- $\operatorname{Re}(z) = (z + z^*)/2$
- $\operatorname{Im}(z) = (z - z^*)/(2i)$