Chapter Notes: Similarity

MATHEMATICS • CLASS 10TH

1. Ratio of Areas of Two Triangles

The area of any triangle is calculated using the fundamental formula:

$$Area = ½ \times base \times height$$

When we find the ratio of the areas of any two triangles, the ratio is equal to the ratio of the product of their bases and corresponding heights. Let ΔABC have base b_1 and height h_1, and ΔPQR have base b_2 and height h_2:

$$A(\Delta ABC) / A(\Delta PQR) = (b_1 \times h_1) / (b_2 \times h_2)$$

Special Cases based on Geometric Conditions:

- **Condition 1 (Equal Heights):** If the heights of both triangles are equal ($h_1 = h_2$), then the ratio of their areas is equal to the ratio of their corresponding bases:

$$A(\Delta ABC) / A(\Delta PQR) = b_1 / b_2$$

- **Condition 2 (Equal Bases):** If the bases of both triangles are equal ($b_1 = b_2$), then the ratio of their areas is equal to the ratio of their corresponding heights:

$$A(\Delta ABC) / A(\Delta PQR) = h_1 / h_2$$

2. Basic Proportionality Theorem (BPT) / Thales Theorem

> **Theorem:** If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

In ΔABC, if a straight line DE is drawn parallel to the side BC intersecting AB at D and AC at E, then according to BPT:

$$AD / DB = AE / EC$$

Converse of Basic Proportionality Theorem:

If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side. That is, if $AD / DB = AE / EC$ in ΔABC, then $DE \parallel BC$.

3. Important Geometric Properties

Property of an Angle Bisector of a Triangle

The bisector of an angle of a triangle divides the side opposite to the angle in the ratio of the remaining sides containing the angle.

In ΔABC, if ray BD is the internal bisector of $\angle ABC$ (where D lies on side AC), then:

$$AB / BC = AD / DC$$

Property of Three Parallel Lines and Their Transversals

The ratio of the intercepts made on a transversal by three parallel lines is equal to the ratio of the corresponding intercepts made on any other transversal by the same parallel lines.

If line $l \parallel \text{line } m \parallel \text{line } n$, and transversal t_1 cuts them at A, B, C while transversal t_2 cuts them at D, E, F, then:

$$AB / BC = DE / EF$$

4. Similarity of Triangles

Two geometric triangles are said to be similar ($\sim$) if their corresponding angles are congruent (equal) and their corresponding sides are in the same proportion.

Axioms / Tests for Similarity:

- **AAA (Angle-Angle-Angle) Test:** If corresponding angles of two triangles are equal, then the triangles are similar. (Note: The **AA Test** is practically used because if two angles of a triangle are equal to two angles of another, the third corresponding angles are automatically equal by the angle sum property).
- **SAS (Side-Angle-Side) Test:** If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, the triangles are similar.
- **SSS (Side-Side-Side) Test:** If all three corresponding sides of two triangles are in the same mathematical proportion, the triangles are similar.

5. Theorem of Areas of Similar Triangles

> **Theorem:** When two triangles are structurally similar, the ratio of the areas of those triangles is equal to the ratio of the squares of their corresponding sides.

If $\Delta ABC \sim \Delta PQR$, then the relationship holds true as follows:

$$A(\Delta ABC) / A(\Delta PQR) = AB^2 / PQ^2 = BC^2 / QR^2 = AC^2 / PR^2$$

💡 Quick Formula Summary Sheet

Concept / Theorem	Mathematical Expression / Ratio
Ratio of Areas (General)	$A_1 / A_2 = (b_1 \times h_1) / (b_2 \times h_2)$
BPT Conditions (when $DE \parallel BC$)	$AD / DB = AE / EC$
Angle Bisector Property	$Side_1 / Side_2 = Segment_1 / Segment_2$
Areas of Similar Triangles	$A_1 / A_2 = (Side_1 / Side_2)^2$